Concerning the Determinant of the Expanded Form of $[A + iB]$

L. L. BUCCIARELLI JR.*
Massachusetts Institute of Technology, Cambridge, Mass.

In the analysis of vibration or stability of elastic systems, one is liable to encounter an eigenvalue problem of the form

$$[A + iB]\{z\} = 0 \tag{1}$$

where the $n \times n$ arrays $[A]$ and/or $[B]$ depend(s) on some real parameter which, when taking on certain characteristic values, enables one to find nontrivial solutions for the complex column $(n \times 1)$, array $\{z\}$.

Letting

$$\{z\} = \{x\} + i\{Y\} \tag{2}$$

the problem may be written

$$\left[\begin{array}{c|c} A & -B \\ \hline B & A \end{array}\right]\left\{\begin{array}{c} X \\ \hline Y \end{array}\right\} = 0 \tag{3}$$

However, if one searches for those characteristic values by evaluating the determinate of the coefficient matrix in Eq. (3), i.e., finding when that determinant passes through zero, one will encounter some difficulty since this determinant is a positive definite quantity. The author ran into this difficulty in an investigation of the torsional stability of shallow shells of revolution.[1]

That this determinant is always positive or zero follows from the consideration of the identity:

$$\left[\begin{array}{c|c} A & -B \\ \hline B & A \end{array}\right]\left[\begin{array}{c|c} I & -iI \\ \hline -iI & I \end{array}\right] = \left[\begin{array}{c|c} I & -iI \\ \hline -iI & I \end{array}\right]\left[\begin{array}{c|c} A + iB & 0 \\ \hline 0 & A - iB \end{array}\right] \tag{4}$$

Noting that the determinant†

$$\text{DET}\left[\begin{array}{c|c} I & -iI \\ \hline -iI & I \end{array}\right] \neq 0$$

and letting

$$\text{DET}\ [A + iB] = D_R + iD_I$$

we have

$$\text{DET}\left[\begin{array}{c|c} A & -B \\ \hline B & A \end{array}\right] = D_R^2 + D_I^2$$

Thus, any numerical evaluation of the determinant of the coefficient matrix in Eq. (3) will not lead to an unambiguous definition of eigenvalues. A better approach to the problem would be to study the behavior of the real and imaginary parts of the determinant of the coefficient matrix in Eq. (1) and determine when these pass through zero.

Reference

[1] Bucciarelli, L. L., Jr., "Torsional Stability of Shallow Shells of Revolution," Ph.D. thesis, June 1963, Massachusetts Institute of Technology, Cambridge, Mass.

Received May 12, 1969.

* Assistant Professor, Department of Aeronautics and Astronautics.

† In fact, this determinant is $(2)^n$.